

Progress on semi-leptonic $B_{(s)}$ decays

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We report on the current status of semi-leptonic $B_{(s)}$ decays, included rare decays, and the extraction of the absolute values of the CKM matrix elements $|V_{cb}|$ and $|V_{ub}|$.

1. Introduction

The values of the CKM matrix elements are not predictable within the Standard Model (SM) and have to be inferred by experimental data. At present, the most precise values of $|V_{cb}|$ and $|V_{ub}|$ come from semi-leptonic decays, driven, at parton level, by the tree level decays $b \rightarrow c(u)l\nu$. The determinations of $|V_{cb}|$ and $|V_{ub}|$ from inclusive and exclusive decays rely on different theoretical calculations, each with different (independent) uncertainties, and on different experimental techniques which have, to a large extent, uncorrelated statistical and systematic uncertainties. This independence makes the comparison of $|V_{cb}|$ and $|V_{ub}|$ values from inclusive and exclusive decays an interesting test of our physical understanding. An indirect determination of $|V_{ub}|$ is also given by the measurement of the rate of the leptonic decays $B^+ \rightarrow l^+\nu$, provided that the B -decay constant is known from theory. This determination is disadvantaged by the helicity suppression and the possibility of a more relevant role of new physics. We summarize significant and recent results on heavy-to-heavy and heavy-to-light semileptonic decays, as well as the actual scenario of $|V_{cb}|$ and $|V_{ub}|$ extraction. We also discuss B mesons semileptonic decays to excited states of the charm meson spectrum. Lastly, we outline the status of rare decays, where hints of new physics have been very recently reported. Reviews on semi-leptonic B decays and on CKM matrix elements extraction are already available^a, but an update seems timely, since progress in data and theory is quickly accumulating.

^asee e.g. 1, 2, 3 and references therein.

1.1. Exclusive heavy-to-heavy decays

In B decays, approximations and techniques of the heavy quark effective theory (HQET) are used, due to the large b -quark mass. In $B \rightarrow D^{(*)}$ semi-leptonic decays, also the mass of the c -quark can be considered large compared to the QCD scale, allowing further approximations.

The form factors depend on $\omega = v_B \cdot v_{D^{(*)}}$, the only scalar formed from the B and $D^{(*)}$ velocities ($v_B^2 = v_{D^{(*)}}^2 = 1$ by definition). The scalar ω is related to q^2 , the momentum transferred to the lepton pair, according to the relation $\omega = (m_B^2 + m_{D^{(*)}}^2 - q^2)/(2m_B m_{D^{(*)}})$. For negligible lepton masses ($l = e, \mu$), the differential ratios can be written as

$$\begin{aligned} \frac{d\Gamma}{d\omega}(B \rightarrow D l \nu) &= \frac{G_F^2}{48\pi^3} (m_B + m_D)^2 m_D^3 (\omega^2 - 1)^{\frac{3}{2}} |V_{cb}|^2 \mathcal{G}^2(\omega) \\ \frac{d\Gamma}{d\omega}(B \rightarrow D^* l \nu) &= \frac{G_F^2}{48\pi^3} (m_B - m_{D^*})^2 m_{D^*}^3 \chi(\omega) (\omega^2 - 1)^{\frac{1}{2}} |V_{cb}|^2 \mathcal{F}^2(\omega) \end{aligned} \quad (1)$$

in terms of a single form factor $\mathcal{G}(\omega)$ and $\mathcal{F}(\omega)$, for $B \rightarrow D l \nu$ and $B \rightarrow D^* l \nu$, respectively. In the HQET limit, both the form factors become^{4,5}

$$\mathcal{G}(1) = \mathcal{F}(1) = 1 \quad (2)$$

at the zero recoil point $\omega = 1$; when $D^{(*)}$ is at rest with respect to B , the light constituents of the initial and final hadrons are not affected by the transition $b \rightarrow c$. For finite values of m_b and m_c , these unity values are altered by perturbative QCD and EW corrections, and by order $1/m_{c,b}^n$ non-perturbative corrections. Schematically

$$\mathcal{F}(1) = \eta_{EW} \eta_A (1 + \delta_{1/m^2} + \dots) \quad (3)$$

where δ_{1/m^2} are power corrections which are suppressed by a factor of at least $\Lambda_{QCD}^2/m_c^2 \sim 3\%$, η_{EW} is the electroweak enhancement factor⁶ and $\eta_A(\alpha_s)$ is a short distance QCD coefficient known at order α_s^2 .^{7,8,9} For \mathcal{F} non-perturbative linear corrections are absent at zero recoil¹⁰ and the leading terms are quadratic in $1/m_{b,c}$. A relation similar to Eq. (3) holds for $\mathcal{G}(1)$, with the addition of linear corrections $\delta_{1/m}$.

For the determination of $|V_{cb}|$, the decay $\bar{B} \rightarrow D^* l \bar{\nu}$ is generally preferred, because of higher experimental rate, and the absence of nonperturbative linear corrections to the form factor \mathcal{F} at zero recoil. The starting point is the experimental fit of the product $|\mathcal{F}(\omega)V_{cb}|$, taken at $\omega \neq 1$ to avoid the kinematic suppression factors. Instead, the theoretical evaluation of the form factor $\mathcal{F}(\omega)$ is, in most cases, performed at $\omega = 1$, where relation (3) holds. This mismatch introduces a dependence on the extrapolation from $\omega \neq 1$ to $\omega = 1$.

The most recent Heavy Flavor Averaging Group (HFAG) experimental fit¹¹ gives

$$|V_{cb} \mathcal{F}(1)| = (35.85 \pm 0.11 \pm 0.44) \times 10^{-3} \quad (4)$$

assuming a form factor parametrization devised in 1998,¹² but rescaled to more recent parameter values. The Belle measurement, with 711 fb^{-1} of data collected,

gives currently the most precise values,¹³ followed by the BaBar global fits,¹⁴ with results in agreement.

The FNAL/MILC collaboration has performed the non perturbative determination of the form factor $\mathcal{F}(1)$ in the lattice unquenched approximation, which includes loops of up, down and strange sea quarks.^{15, 16} The up and down quarks are usually taken to be degenerate, so those simulations are referred to as $n_f = 2 + 1$. There is a very recent update that uses the full suite of MILC (2+1)-flavor asqtad ensembles with lattice spacings as small as 0.045 fm and light-to-strange-quark mass ratios as low as 1/20,¹⁷ giving

$$\mathcal{F}(1) = 0.906 \pm 0.004 \pm 0.012 \quad (5)$$

without the EW enhancement factor η_{EW} . The first error is statistical and the second one systematic. The estimate for $|V_{cb}|$, using the latest HFAG average, is reported in Table (1). The QCD error is now commensurate with the experimental error. At the current level of precision, it would be important to extend the calculation to nonzero recoil. Indeed, at finite momentum transfer, only old quenched lattice results are available;¹⁸ which, combined with 2008 BaBar data,¹⁴ give the $|V_{cb}|$ value also reported in table (1).

The lattice calculations have to be compared with non-lattice ones. The more recent value obtained by using zero recoil sum rules reads^{19, 20}

$$\mathcal{F}(1) = 0.86 \pm 0.02 \quad (6)$$

Full α_s and up to $1/m^3$ corrections have been included; the impact of $1/m^4$ and $1/m^5$ corrections has been estimated. A recent parameter update related to power corrections does not affect significantly the results.²¹ The corresponding estimate of $|V_{cb}|$, using the HFAG average in Eq. (4), is reported in Table (1). The slightly smaller values for the form factors in sum rules determinations imply slightly higher values of V_{cb} . The theoretical error is more than twice the error in the lattice determinations.

Let us now consider $B \rightarrow D l \nu$ decays. The most recent Heavy Flavor Averaging Group (HFAG) experimental fit, adopting the same parametrization used for $B \rightarrow D^* l \nu$ decays, gives¹¹

$$|V_{cb} \mathcal{G}(1)| = (42.64 \pm 0.72 \pm 1.35) \times 10^{-3} \quad (7)$$

Recent progress has been reported by the FNAL/MILC collaboration,²² updating the values for the form factor at zero recoil in the unquenched form approximation, dating back to 2005-06.^{23, 24} The propagating heavy quarks on the lattice have been interpreted by means of an effective theory approach. The hadronic form factors have been computed in the unquenched approximation as well. The related estimate of $|V_{cb}|$ produced at nonzero recoil in the joint fit with the 2009 Babar data,²⁵ is reported in Table (1). To quantify the improvement due to working at nonzero recoil, $|V_{cb}|$ is extracted by extrapolating the experimental data to zero recoil and compared

Table 1. Exclusive $|V_{cb}|$ determinations

<i>Exclusive decay</i>	$ V_{cb} \times 10^3$
<i>$B \rightarrow D^* l \bar{\nu}$</i>	
FNAL/MILC (Lattice unquenched) ¹⁷	$39.04 \pm 0.49_{\text{exp}} \pm 0.53_{\text{QCD}} \pm 0.19_{\text{QED}}$
HFAG (Lattice unquenched) ^{11, 15, 16}	$39.54 \pm 0.50_{\text{exp}} \pm 0.74_{\text{th}}$
Rome (Lattice quenched $\omega \neq 1$) ^{18, 14}	$37.4 \pm 0.5_{\text{exp}} \pm 0.8_{\text{th}}$
HFAG (Sum Rules) ^{11, 19, 20}	$41.6 \pm 0.6_{\text{exp}} \pm 1.9_{\text{th}}$
<i>$B \rightarrow D l \bar{\nu}$</i>	
FNAL/MILC (Lattice unquenched $\omega \neq 1$) ^{22, 25}	$38.5 \pm 1.9_{\text{exp+lat}} \pm 0.2_{\text{QED}}$
PDG (HQE + BPS) ^{29, 28}	$40.6 \pm 1.5_{\text{exp}} \pm 0.8_{\text{th}}$
Rome (Lattice quenched $\omega \neq 1$) ^{25, 26}	$41.6 \pm 1.8_{\text{stat}} \pm 1.4_{\text{syst}} \pm 0.7_{\text{FF}}$

with the theoretical form factor at that point. The result is found consistent with the nonzero recoil determination, with an error 25% larger.²²

The previous lattice determination of $|V_{cb}|$ at non-zero recoil was given in the quenched approximation.^{26, 27} It was based on the step scaling method, which avoids the recourse to HQET. The related estimate of $|V_{cb}|$ by the Babar Collaboration,²⁵ with the same 2009 data, gives an higher value, reported in Table (1) (the errors are statistical, systematic and due to the theoretical uncertainty in the form factor \mathcal{G} , respectively).

The latest results from non-lattice calculations are about 10 years old, and use the expansion around the "BPS" limit, that is the limit where the parameters related to kinetic energy and the chromomagnetic moment are equal in the heavy quark expansion.²⁸ Under these assumptions, the Particle Data Group finds the form factor²⁹

$$\mathcal{G}(1) = 1.04 \pm 0.02 \quad (8)$$

and the related $|V_{cb}|$ value reported in Table (1).

Data on $B^0 \rightarrow D^{(*)+} \mu^- \nu$ decays have been provided until now by electron-positron machines, especially the dedicated B -Factories BaBar and Belle. At LHCb, statistics is accumulating and about 5 million $B^0 \rightarrow D^{(*)+} \mu^- \nu$ decays are available.³⁰ At hadron colliders semi-leptonic decays have an high branching ratio, and muons in the final state allow an easy triggering. However, the measurements of $|V_{cb}|$ and $|V_{ub}|$ imply the reconstruction, in the b -hadron rest frame, of observables difficult to measure at hadron colliders, such as the squared invariant mass of the lepton pair q^2 . At LHCb, it is possible to improve the q^2 resolution by exploiting the separation between primary and secondary vertices, determining the B flight direction vector and measuring the neutrino momentum with a two-fold ambiguity.³¹

The $B \rightarrow D^{(*)} \tau \nu_\tau$ decays are more difficult to measure, since decays into the heaviest τ lepton are suppressed and there are multiple neutrinos in the final state, following the τ decay. Multiple neutrinos stand in the way of the reconstruction

of the invariant mass of B meson, and additional constraints related to the B production are required. At the B factories, a major constraint exploited is the fact that B mesons are produced from the process $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B\bar{B}$.

The latest results belong to the BaBar Collaboration, that has measured the $\bar{B} \rightarrow D^{(*)}\tau^-\bar{\nu}_\tau$ branching fractions normalized to the corresponding $\bar{B} \rightarrow D^{(*)}l^-\bar{\nu}_l$ modes (with $l = e, \mu$) by using the full BaBar data sample, and found^{32,33}

$$\begin{aligned}\mathcal{R}_{\tau/l}^* &\equiv \frac{\mathcal{B}(\bar{B} \rightarrow D^*\tau^-\bar{\nu}_\tau)}{\mathcal{B}(\bar{B} \rightarrow D^*l^-\bar{\nu}_l)} = 0.332 \pm 0.024 \pm 0.018 \\ \mathcal{R}_{\tau/l} &\equiv \frac{\mathcal{B}(\bar{B} \rightarrow D\tau^-\bar{\nu}_\tau)}{\mathcal{B}(\bar{B} \rightarrow Dl^-\bar{\nu}_l)} = 0.440 \pm 0.058 \pm 0.042\end{aligned}\tag{9}$$

where the first uncertainty is statistical and the second is systematic. The results exceed the SM predictions $\mathcal{R}_{\tau/l}^*(SM) = 0.252 \pm 0.003$ and $\mathcal{R}_{\tau/l}(SM) = 0.297 \pm 0.017$ by 2.7σ and 2.0σ , respectively. The combined significance of this disagreement is 3.4σ .^{32,33} Other estimates give $\mathcal{R}_{\tau/l}(SM) = 0.31 \pm 0.02$, with a combined phenomenological and lattice analysis,³⁴ and a similar result, $\mathcal{R}_{\tau/l}(SM) = 0.316 \pm 0.012 \pm 0.007$, where the errors are statistical and total systematic, respectively, is found in a (2+1)-flavor lattice QCD calculation.³⁵ Both analysis reduce the significance of the discrepancy for $\mathcal{R}_{\tau/l}$ below 2σ .

The BaBar results (9) are in agreement (with smaller uncertainties) with measurements by Belle using the $\Upsilon(4S)$ data set that corresponds to an integrated luminosity of 605 fb^{-1} and contains 657×10^6 $B\bar{B}$ events.³⁶ The branching ratio measured values have consistently exceeded the SM expectations with increased precision, that starts to be enough to constrain NP. Latest data from BaBar are not compatible with a charged Higgs boson in the type II two-Higgs-doublet model and with large portions of the more general type III two-Higgs-doublet model.³³ Several NP frameworks have been studied that explain (or fail to) this alleged hint of breaking of lepton-flavour universality. Minimal flavor violating models, right-right vector and right-left scalar quark currents, leptoquarks, quark and lepton compositeness models have been investigated,^{37,38} as well as modified $Wl\nu$ couplings in the SM extended by additional sterile neutrinos,³⁹ the possibility of an additional tensor operator in the effective weak hamiltonian,⁴⁰ charged scalar contributions,⁴¹ aligned two Higgs doublet models,⁴² effective Lagrangians,^{43,44} new sources of CP violation,⁴⁵ and so on.

There is room for improvement in current statistic limits for measurements of $\mathcal{R}_{\tau/l}$. It would be interesting to investigate if the results of the Belle analysis shift towards the SM results, obtained by global unitarity-triangle fits, by using the full $\Upsilon(4S)$ data sample containing 772×10^6 $B\bar{B}$ pairs and the improved hadronic tagging, as happened in the case of purely leptonic decays $B^- \rightarrow \tau^-\bar{\nu}_\tau$.⁴⁶

The Belle-II experiment will have an integrated luminosity over 30 times greater than that of the combined BaBar and Belle datasets. It should start in 2016, and in the next decade provide accurate measurements to investigate possible NP contributions. The estimate error with 75 ab^{-1} at the $\Upsilon(4S)$ is around 1%.

In this section, we have always implicitly alluded to B decays, but semi-leptonic B_s decays can also probe CKM matrix elements. Moreover, semi-leptonic B_s^0 decays are used as a normalization mode for various searches for new physics at hadron colliders and at Belle-II. The presence of the heavier spectator strange quark is bound to introduce some amount of SU(3) symmetry breaking, which may affect width ratios^{47,48} of the kind of

$$\frac{\Gamma(B_s \rightarrow X l \nu)}{\Gamma(B_d \rightarrow X l \nu)} \quad (10)$$

The branching fractions of $B_s \rightarrow X l \nu$ decays, where X is an arbitrary final state, have been measured at BaBar⁴⁹ and Belle,⁵⁰ in data-sets obtained from energy scans above the $\Upsilon(4S)$. Belle has profited of the large available data sample of 121 fb^{-1} near the $\Upsilon(5S)$ resonance to perform the most precise measurement of the branching fraction⁵⁰

$$\mathcal{B}(B_s \rightarrow X l \nu) = [10.6 \pm 0.5_{\text{stat}} \pm 0.7_{\text{syst}}]\% \quad (11)$$

Other measurements of B_s branching ratios have also been reported, precisely of $B_s^0 \rightarrow D_{s1}^-(2536)\mu^+\nu X$ decays by DØ⁵¹ and of $\bar{B}_s^0 \rightarrow D_{s2}^{*+}(2573)\mu^-\bar{\nu}X$ decays by LHCb.⁵²

Outside the $\Upsilon(4S)$ region, the determination of $|V_{cb}|$ is also possible by studying the baryonic $\Lambda_b^0 \rightarrow \Lambda_c^+ l^- \bar{\nu}$ decays.⁵³

2. B -Mesons Decays to Excited D -Meson States

The increased interest in semi-leptonic B decays to excited states of the charm meson spectrum derives by the fact that they contribute as a background to the direct decay $B^0 \rightarrow D^* l \nu$ at the B factories, and, as a consequence, as a source of systematic error in the $|V_{cb}|$ measurements.

The spectrum of mesons consisting of a charm and an up or a down anti-quark is poorly known. In the non-relativistic constituent quark model, the open charm system can be classified according to the radial quantum number and to the eigenvalue L of the relative angular momentum between the c-quark and the light degrees of freedom. The low-mass spectrum is comprised of the ground states ($1S$, $L = 0$), that is D and D^* mesons, the orbital excitations with angular momentum $L = 1, 2$ ($1P$, $1D$), and the first radial excitations ($2S$).

The four states with $L = 1$ are generically denoted as D^{**b} . and have been identified as $D_0^*(2400)$, $D_1(2420)$, $D_1'(2430)$ (or $D_1(2430)$), and $D_2^*(2460)$.²⁹ Two of them, $D_1(2420)$ and $D_2^*(2460)$, have relatively narrow widths, about 20-30 MeV, and have been observed and studied by a number of experiments since the nineties (see Ref. 54 and Refs. therein). The other two states, $D_0^*(2400)$, $D_1'(2430)$, are more

^bSometimes in literature this term is extended to include all particles in the low-mass spectrum except the ground states.

difficult to detect due to the large width, about 200-400 MeV, and their observation has started more recently.^{55, 56, 57, 58, 59}

In 2010 BaBar has observed, for the first time, candidates for the radial excitations of the D^0 , D^{*0} and D^{*+} , as well as the $L = 2$ excited states of the D^0 and D^+ .⁶⁰ Resonances in the 2.4-2.8 GeV/ c^2 region of hadronic masses have also been identified at LHCb.⁶¹ In the same region, data are available on semi-leptonic B decays to final states containing a $D_s^{(*)+}K$ system.^{62, 63}

The not completely clear experimental situation is mirrored by two theoretical puzzles. Most calculations, using sum rules,^{64, 65} quark models,^{66, 67, 68} OPE^{69, 70} (but not constituent quark models⁷¹), indicate that the narrow width states dominate over the broad D^{**} states, in contrast to experiments (the “1/2 vs 3/2 puzzle”). One possible weakness common to these theoretical approaches is that they are derived in the heavy quark limit and corrections might be large. The other puzzle is that the sum of the measured semi-leptonic exclusive rates having $D^{(*)}$ in the final state is less than the inclusive one (“gap problem”).^{59, 72} To overcome the difficulties to disentangle very broad resonances from continuum, both on theoretical and experimental sides, it has been suggested to clarify the comparison between theory and experiment analyzing states analogous to D_0^* and D_1' , but narrow, in particular studying the decay $B_s^0 \rightarrow \bar{D}_{sJ}\pi$.⁷³ Other theoretical suggestions to ease or solve the previous problems include taking into account an unexpectedly large B decay rate to the first radially excited $D'^{(*)}$.^{74, 20} A recent proposal is to extract exclusive branching fractions of semi-leptonic B-meson decays to charmed mesons from a fit to electron energy, hadronic mass and combined hadronic mass-energy momenta measured in inclusive $B \rightarrow X_c l \nu$ decays, as an alternative to direct measurements.⁷⁵

Recently, first dynamical lattice computations of the $\bar{B} \rightarrow D^{**} l \nu$ form factors have been attempted, although still preliminary.^{76, 77}

2.1. Inclusive $B \rightarrow X_c l \nu_l$ decays

In inclusive $B \rightarrow X_c l \nu_l$ decays, the final state X_c is an hadronic state originated by the charm quark. There is no dependence on the details of the final state, and quark-hadron duality is generally assumed. Long distance dynamics of the meson can be factorized by using an OPE approach, which, combined with HQET, gives to inclusive transition rates the form of an (Heavy Quark) Expansion (HQE) in $1/m_b$. The coefficients of the expansion are calculable in perturbation theory, while the hadronic expectation values of the operators encode the nonperturbative corrections and depend on a number of HQE parameters, which increase at increasing powers of $1/m_b$. These parameters are affected by the particular theoretical framework (scheme) that is used to define the quark masses. The HQE is valid only for sufficiently inclusive measurements, therefore the relevant quantities to be measured are global shape parameters (the first few moments of various kinematic distributions) and the total rate.

At order $1/m_b^0$, the parton level, we have the usual α_s expansion, which is known

Table 2. $|V_{cb}|$ inclusive determinations

<i>Inclusive decays</i>	$ V_{cb} \times 10^3$
global fit, kin scheme, m_c constraint (HFAG) ¹¹	$41.88 \pm 0.44_{\text{fit}} \pm 0.59_{\text{th}}$
global fit, kin scheme, $B \rightarrow X_s \gamma$ constraint (HFAG) ¹¹	$41.94 \pm 0.43_{\text{fit}} \pm 0.59_{\text{th}}$
global fit, kin scheme, m_c constraint ²¹	42.42 ± 0.86

completely to order α_s and α_s^2 , for the width and moments of the lepton energy and hadronic mass distributions (see Refs. 78, 79, 80, 81 and references therein). The terms of order $\alpha_s^{n+1}\beta_0^n$, where β_0 is the first coefficient of the QCD β function, have also been computed following the BLM procedure.^{82,78}

At the next order in the HQE, that is Λ_{QCD}^2/m_b^2 , there are two operators, called the kinetic energy and the chromomagnetic operator. The perturbative corrections to the coefficient of the kinetic matrix element have been evaluated at order α_s for generic observables, such as partial rates and moments.^{83,84} They lead to numerically modest modifications of the width and moments. Corrections at order α_s to the coefficient of the matrix element of the chromomagnetic operator have also been computed.^{85,86} The results have been employed to evaluate the correction to the semi-leptonic decay width, the mean lepton energy, and the variance (second central moment) of the lepton energy distribution. The complete corrections of the $\alpha_s \Lambda_{QCD}^2/m_b^2$ to the width is a few per mill on the width, but the corrections to the first two leptonic moments are of the same order of the experimental errors.⁸⁶ The estimate of these effects on the determination of $|V_{cb}|$ is under the way.

Neglecting perturbative corrections, i.e. working at tree level, contributions to various observables have been computed at order $1/m_b^3$ and estimated at order $1/m_b^{4,5}$, $1/m_b^3 m_c^2$; contributions at order $\alpha_s(m_c)1/m_b^3 m_c$ and the so-called intrinsic charm term have been estimated as well.^{87,88,89,90,91,92}

A global fit is a simultaneous fit to HQE parameters, quark masses and absolute values of CKM matrix elements obtained by measuring spectra plus all available moments. Only the HQE parameters associated with $O(1/m_b^{2,3})$ corrections are routinely fitted from experiments, one reason being the growth in their number at higher orders. The HFAG global fit employs as experimental inputs the (truncated) moments of the lepton energy E_l (in the B rest frame) and the m_X^2 spectra in $B \rightarrow X_c l \nu$.¹¹ The results, although sufficient for determining $|V_{cb}|$, measure the b -quark mass only to about 50 MeV precision. To get higher precision, additional constraints are introduced: the photon energy moments in $B \rightarrow X_s \gamma$, or a precise constraint on the c -quark mass. The actual HFAG global fit is performed in the kinetic scheme and yields

$$|V_{cb}| = (41.88 \pm 0.44_{\text{fit}} \pm 0.59_{\text{th}}) \times 10^{-3} \quad (12)$$

with the constrained value $m_c^{\overline{\text{MS}}}(3\text{GeV}) = (0.998 \pm 0.029) \text{ GeV}$, obtained using

low-energy sum rules.⁹³ By using the $B \rightarrow X_s \gamma$ constraints, it gives

$$|V_{cb}| = (41.94 \pm 0.43_{\text{fit}} \pm 0.59_{\text{th}}) \times 10^{-3} \quad (13)$$

The precision is higher than in the exclusive determinations, being about 1.7%. Another recent determination in the kinetic scheme of $|V_{cb}|$ gives²¹

$$|V_{cb}| = (42.42 \pm 0.86) \times 10^{-3} \quad (14)$$

In analogy to the HFAG determination, it uses the full $O(\alpha_s^2/m_b^0)$ calculation and no $O(\alpha_s/m_b^2)$ calculations, but it employs a slightly different constraint on m_c . The error is also slight larger, about 2%, comparable with the best errors of the exclusive determination. The results have been collected in Table 2, that, compared with the results in Table 1 for the exclusive determination, shows a tension in most cases around 2-3 σ , the precise number depending on the values used for the comparison. One could also compare with indirect fits, provided by the UTfit collaboration⁹⁴

$$|V_{cb}| = (42.12 \pm 0.07) \times 10^{-3} \quad (15)$$

and by the CKMfitter collaboration (at 1 σ).⁹⁵

$$|V_{cb}| = (41.51^{+0.56}_{-1.15}) \times 10^{-3} \quad (16)$$

Indirect fits prefer a value for $|V_{cb}|$ that is closer to the (higher) inclusive determination.

High statistic B -factories have greatly contributed to the increase in measurement precision with respect to previous experiments, and the high statistics at Belle II at SuperKEKB is expected, within the next decade, to push errors on $|V_{cb}|$ down to 1%.⁹⁶

2.2. Exclusive heavy-to-light decays

The analysis of exclusive charmless semi-leptonic decays, in particular the $\bar{B} \rightarrow \pi l \bar{\nu}_l$ decay, is currently employed to determine the CKM parameter $|V_{ub}|$. The $B \rightarrow \pi l \nu$ decays depend on a single form factor $f_+(q^2)$, in the limit of zero leptonic masses. The first lattice determinations of $f_+(q^2)$ based on unquenched simulations have been obtained by the HPQCD collaboration⁹⁷ and the Fermilab/MILC collaboration;⁹⁸ they are in substantial agreement. These analyses, at $q^2 > 16 \text{ GeV}^2$, together with latest data on $B \rightarrow \pi l \nu$ decays coming from Belle and BaBar, and 2007 data from CLEO, have been employed in the actual HFAG averages.¹¹ The results have been reported in Table 3. Also, HFAG has performed a simultaneous fit of the BCL parametrization⁹⁹ to lattice results and experimental data, to exploit all the available information in the full q^2 range, which has given the following average value

$$|V_{ub}| = (3.28 \pm 0.29) \times 10^{-3} \quad (17)$$

On the lattice front, the Fermilab/MILC collaboration has recently presented an update, a blinded analysis over a range of lattice spacings $a \sim 0.045\text{-}0.12 \text{ fm}$.¹⁰²

Table 3. $|V_{ub}|$ exclusive determinations

<i>Exclusive decays</i>	$ V_{ub} \times 10^3$
$\bar{B} \rightarrow \pi l \bar{\nu}_l$	
HPQCD ($q^2 > 16$) (HFAG) ^{97, 11}	$3.52 \pm 0.08^{0.61}_{0.40}$
Fermilab/MILC ($q^2 > 16$) (HFAG) ^{98, 11}	$3.36 \pm 0.08^{0.37}_{0.31}$
lattice, full q^2 range (HFAG) ¹¹	3.28 ± 0.29
LCSR ($q^2 < 12$) (HFAG) ^{100, 11}	$3.41 \pm 0.06^{+0.37}_{-0.32}$
LCSR ($q^2 < 16$) (HFAG) ^{101, 11}	$3.58 \pm 0.06^{+0.59}_{-0.40}$

Preliminary results have been presented by the ALPHA^{103, 104} ($n_f = 2$) HPQCD¹⁰⁵ ($n_f = 2 + 1$), and the RBC/UKQCD¹⁰⁶ ($n_f = 2 + 1$) Collaborations. Recent results are also available on a fine lattice (lattice spacing $a \sim 0.04$ fm) in the quenched approximations by the QCDSF collaboration.¹⁰⁷

In the complementary kinematic region, at large recoil, direct LCSR calculations of the semi-leptonic form factors are available, which have benefited by progress in pion distribution amplitudes, next-to-leading (NLO) and leading (LO) higher order twists (see e.g.^{100, 108, 109} and Refs. within). The $|V_{ub}|$ estimate are generally higher than the corresponding lattice ones, but still in agreement, within the relatively larger theoretical errors. The estimated values for $|V_{ub}|$ according to LCSR^{101, 100} provided by HFAG have been reported in Table 3. Higher values for $|V_{ub}|$ have been computed in the relativistic quark model.¹¹⁰

Recently, significantly improved branching ratios of other heavy-to-light semi-leptonic decays have been reported, that reflect on increased precision for $|V_{ub}|$ values inferred by these decays. $|V_{ub}|$ has been extracted from $B^+ \rightarrow \omega l^+ \nu$,¹¹¹ yielding, with the LCSR form factor determination¹¹²

$$|V_{ub}| = (3.41 \pm 0.31) \times 10^{-3} \quad (18)$$

and, with the ISGW2 quark model¹¹³

$$|V_{ub}| = (3.43 \pm 0.31) \times 10^{-3} \quad (19)$$

A major problem is that the quoted uncertainty does not include any uncertainty from theory, since uncertainty estimates of the form-factor integrals are not available.

Other channels to have been investigated are $B \rightarrow \rho l \nu$ decays. By comparing the measured distribution in q^2 , with an upper limit at $q^2 = 16$ GeV, for $B \rightarrow \rho l \nu$ decays, with LCSR predictions for the form factors,¹¹² the $|V_{ub}|$ value reads¹¹⁴

$$|V_{ub}| = (2.75 \pm 0.24) \times 10^{-3} \quad (20)$$

and with the ISGW2 quark model.¹¹³

$$|V_{ub}| = (2.83 \pm 0.24) \times 10^{-3} \quad (21)$$

Other interesting channels are $B \rightarrow \eta^{(\prime)} l \nu$ decays, not yet sufficiently precise to be used for $|V_{ub}|$ extraction. The value of the ratio

$$\frac{\mathcal{B}(B^+ \rightarrow \eta' l^+ \nu_l)}{\mathcal{B}(B^+ \rightarrow \eta l^+ \nu_l)} = 0.67 \pm 0.24_{\text{stat}} \pm 0.11_{\text{syst}} \quad (22)$$

seems to allow an important gluonic singlet contribution to the η' form factor.^{115, 116}

In future prospects, other channels that can be valuable to extract $|V_{ub}|$ are $B_s \rightarrow K^{(*)} \bar{l} \nu$ decays.¹¹⁷ Other semileptonic decays, as $B^- \rightarrow \pi^+ \pi^- l^- \bar{\nu}_l$, through the analysis of their angular variables, can be used to measure dipion form factors.¹¹⁸

Baryonic semi-leptonic decays are the subject of a growing interest,⁵³ in particular the $|V_{ub}|$ sensitive $\Lambda_b^0 \rightarrow p l^- \bar{\nu}$ decays, whose form factors have been evaluated in the LCSR¹¹⁹ and in the lattice with static b -quarks frameworks.¹²⁰ Help in constraining the baryonic transition form factor in B decays may instead come from the recent evidence for the semi-leptonic decay $B^- \rightarrow p \bar{p} l^- \bar{\nu}_l$ ($l = e, \mu$).¹²¹

The purely leptonic decay $B^- \rightarrow \tau^- \bar{\nu}_\tau$, first observed by Belle in 2006,¹²² has the SM branching ratio

$$\mathcal{B}(B^- \rightarrow \tau^- \bar{\nu}_\tau) = \frac{G_F^2 m_B m_\tau^2}{8\pi} \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 f_B^2 |V_{ub}|^2 \tau_B \quad (23)$$

and its measurement provides a direct experimental determination of the product $f_B |V_{ub}|$. The new Belle result⁴⁶

$$\mathcal{B}(B^- \rightarrow \tau^- \bar{\nu}_\tau) = (0.72_{-0.25}^{+0.27} \pm 0.11) \times 10^{-4} \quad (24)$$

where the first errors are statistical and the second ones systematical, differs from previous BaBar and Belle results, which, averaged by HFAG,¹¹ give the branching fraction

$$\mathcal{B}(B^- \rightarrow \tau^- \bar{\nu}_\tau) = (1.67 \pm 0.30) \times 10^{-4} \quad (25)$$

By using the recent Belle value together with the previous Belle measurement based on a semi-leptonic B tagging method and taking into account all the correlated systematic errors, the branching fraction is found to be⁴⁶

$$\mathcal{B}(B^- \rightarrow \tau^- \bar{\nu}_\tau) = (0.96 \pm 0.26) \times 10^{-4} \quad (26)$$

Combining this value with the mean B^+ -meson lifetime $\tau_B = 1.641 \pm 0.008$ ²⁹ and their average for the B meson decay constant, $f_B = 190.5 \pm 4.2$ MeV ($n_f = 2 + 1$), the FLAG (Flavor Lattice Averaging Group) collaboration obtains¹²³

$$|V_{ub}| = (3.87 \pm 0.52 \pm 0.09) \times 10^{-3} \quad (27)$$

where the first error comes from experiment and the second comes from the uncertainty in f_B . The accuracy is not yet enough to make this channel competitive for $|V_{ub}|$ extraction. In contrast with previous experimental analyses, the new Belle

data seem to indicate agreement with previous results from the SM. Search of possible lepton flavour violations can also be made independent of $|V_{ub}|$ by building ratios of branching fractions, such as

$$R' = \frac{\tau_{B^0}}{\tau_{B^+}} \frac{\mathcal{B}(B^+ \rightarrow \tau^+ \nu_\tau)}{\mathcal{B}(B^0 \rightarrow \pi^- l^+ \nu_l)} \quad (28)$$

2.3. Inclusive $B \rightarrow X_u l \nu_l$ decays

The extraction of $|V_{ub}|$ from inclusive decays requires to address theoretical issues absent in the inclusive $|V_{cb}|$ determination. OPE techniques are not applicable in the so-called endpoint or singularity or threshold phase space region, corresponding to the kinematic region near the limits of both the lepton energy E_l and q^2 phase space, where the rate is dominated by the production of low mass final hadronic states. This region is plagued by the presence of large double (Sudakov-like) perturbative logarithms at all orders in the strong coupling. Corrections can be large and need to be resummed at all orders^c. The kinematics cuts due to the large $B \rightarrow X_c l \nu$ background enhance the weight of the threshold region with respect to the case of $b \rightarrow c$ semi-leptonic decays; moreover, in the latter, corrections are not expected as singular as in the $b \rightarrow u$ case, being cutoff by the charm mass.

On the experimental side, efforts have been made to control the background and access to a large part of the phase space, so as to reduce, on the whole, the weight of the endpoint region. Latest results by Belle¹³² and BaBar¹³³ use their complete data sample, 657×10^6 $B\text{-}\bar{B}$ pairs for Belle and 467×10^6 $B\text{-}\bar{B}$ pairs for BaBar. Although the two analyses differ in the treatment of the background, both collaborations claim to access $\sim 90\%$ of the phase space.

On the theoretical side, several schemes are available. All of them are tailored to analyze data in the threshold region, but differ significantly in their treatment of perturbative corrections and the parametrization of non-perturbative effects.

The average values for $|V_{ub}|$ have been extracted by HFAG from the partial branching fractions, adopting a specific theoretical framework and taking into account correlations among the various measurements and theoretical uncertainties.¹¹ BaBar and Belle analysis, Refs. 132 and 133, as well as the HFAG averages in Ref. 11 rely on at least four different QCD calculations of the partial decay rate: BLNP by Bosch, Lange, Neubert, and Paz;^{134, 135, 136} DGE, the dressed gluon exponentiation, by Andersen and Gardi;¹³⁷ ADFR by Aglietti, Di Lodovico, Ferrara, and Ricciardi;^{138, 139, 140} and GGOU by Gambino, Giordano, Ossola and Uraltsev.¹⁴¹ These QCD theoretical calculations are the ones taking into account the whole set of experimental results, or most of it, starting from 2002 CLEO data.¹⁴² They can be roughly divided into approaches based on the estimation of the shape function (BLN, GGOU) and on resummed perturbative QCD (DGE, ADFR). Other theoretical schemes have been described in Refs. 143, 144, 145. Although conceptually

^cSee e.g. 124, 125, 126, 127, 128, 129, 130, 131, and references therein.

Table 4. $|V_{ub}|$ inclusive determinations

	<i>Inclusive decays</i> ($ V_{ub} \times 10^3$)			
	BNLP ^{134, 135, 136}	GGOU ¹⁴¹	ADFR ^{138, 139, 140}	DGE ¹³⁷
BaBar ¹³³	$4.28 \pm 0.24^{+0.18}_{-0.20}$	$4.35 \pm 0.24^{+0.09}_{-0.10}$	$4.29 \pm 0.24^{+0.18}_{-0.19}$	$4.40 \pm 0.24^{+0.12}_{-0.13}$
Belle ¹³²	$4.47 \pm 0.27^{+0.19}_{-0.21}$	$4.54 \pm 0.27^{+0.10}_{-0.11}$	$4.48 \pm 0.30^{+0.19}_{-0.19}$	$4.60 \pm 0.27^{+0.11}_{-0.13}$
HFAG 11	$4.40 \pm 0.15^{+0.19}_{-0.21}$	$4.39 \pm 0.15^{+0.12}_{-0.20}$	$4.03 \pm 0.13^{+0.18}_{-0.12}$	$4.45 \pm 0.15^{+0.15}_{-0.16}$

quite different, all the above approaches generally lead to roughly consistent results when the same inputs are used and the theoretical errors are taken into account. The HFAG estimates,¹¹ together with the latest estimates by BaBar and Belle,^{132, 133} are reported in Table 4,

We can also compare with indirect fits

$$|V_{ub}| = (3.61 \pm 0.12) \times 10^{-3} \quad (29)$$

by UTfit⁹⁴ and

$$|V_{ub}| = (3.49^{+0.21}_{-0.10}) \times 10^{-3} \quad (30)$$

at 1σ by CKMfitter.⁹⁵ At variance with the $|V_{cb}|$ case, the results of the global fit prefer a value for $|V_{ub}|$ that is closer to the (lower) exclusive determination.

3. Rare decays

The increased luminosity of the actual experimental facilities and the possibility to explore rare decays in quantitative detail have prompted a lot of recent theoretical activity.

In the inclusive $B \rightarrow X_s l^+ l^-$ decays the major theoretical uncertainties come from the non-perturbative nature of the intermediate $\bar{c}c$ states. By cutting on the invariant di-lepton mass around the masses of the J/ψ and ψ' resonances, rather precise determinations seem to be possible, since below or above the $\bar{c}c$ resonances, the inclusive decay is dominated by perturbative contributions. The calculations of the perturbative contribution has been extended to the next-to-next-to leading order (NNLO) (see Ref. 146 and Refs. within) greatly reducing the theoretical uncertainty, in particular the matching scale uncertainty at NLO. In the case of inclusive $B \rightarrow X_d l^+ l^-$, the short distance analysis is similar, once one keeps the CKM suppressed terms in the operator expansion. The $b \rightarrow dl^+ l^-$ transition has been investigated in the channel $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ by the LHCb Collaboration, searching for Majorana neutrinos using 3 fb^{-1} of data.¹⁴⁷ The predicted SM branching ratio is of order 10^{-8} .

The inclusive $B \rightarrow X_s l^+ l^-$ decays have been measured at Belle¹⁴⁸ and at BaBar.¹⁴⁹ Both results find branching ratios of order 10^{-6} . Using a sum over exclusive modes as the basis for extrapolation to the fully inclusive rate, a lepton-flavor-averaged inclusive branching fraction has been recently measured by BaBar.¹⁵⁰ Also

the lepton forward-backward asymmetry (assuming that it does not depend on the lepton flavor) has been recently measured, for the first time, by Belle¹⁵¹ in inclusive $B \rightarrow X_s l^+ l^-$. The lepton forward-backward asymmetry had already been measured in exclusive $B \rightarrow K^* l^+ l^-$ channels.^{152, 153, 154, 155}

The NLO and NNLO QCD corrections for inclusive decays can of course be also used for the corresponding exclusive decays. The kinematic available phase space in $B \rightarrow K^{(*)} \mu^+ \mu^-$ is fully covered experimentally, with the exception of the J/ψ and ψ' resonances, which are removed by cuts. In the exclusive channel $B \rightarrow K^* l^+ l^-$, a systematic theoretical description using QCD factorization in the heavy quark limit is possible for small invariant di-lepton masses q^2 , reducing the number of independent form factors from 7 to 2 and allowing to calculate spectator effects.¹⁵⁶ In the region of low q^2 , where the energy of the emitted meson is large in the B meson rest frame, both LCSR and soft collinear effective theory analyses have been performed, with some discrepancy in a form factor ratio.¹⁵⁷ In the high q^2 region, QCD factorization is less justified, becoming invalid close to the endpoint of the spectrum at $q^2 = (m_B - m_K)^2$. Alternative approaches have been developed, based on expansions whose scale is set by the large value of q^2 .^{158, 159, 160, 161} The large q^2 region is the domain of election for lattice QCD and unquenched calculations of form factors have been performed, see e.g..^{162, 163} The more recent ones are for $B \rightarrow K l l$ decays by HPQCD,^{164, 165} for $B_s \rightarrow K l \nu$ and $B \rightarrow K l l$ decays by Fermilab/MILC collaboration¹⁶⁶ and for $B_s \rightarrow \phi l^+ l^-$ and $B \rightarrow K^* l^+ l^-$ decays by the Cambridge collaboration.^{167, 168} In the same large q^2 region, ratios of $B \rightarrow K^*$ form factors have been extracted from angular variables recently measured,^{154, 169, 170} precisely the fraction of longitudinally polarized vector mesons and the transverse asymmetry in the $B \rightarrow K^* l^+ l^-$ decay, and found consistent with lattice results.¹⁷¹ In general, the study of angular observables can be used advantageously in the $B \rightarrow K^* l^+ l^-$ decay, even to explore the possibility of new physics.^{172, 173, 174, 175, 176, 177} The angular distribution $B \rightarrow K^*(\rightarrow K\pi) l^+ l^-$ may be polluted by events coming from the distribution $B \rightarrow K_0^*(\rightarrow K\pi) l^+ l^-$, where K_0^* is a scalar meson resonance, and this possibility was analyzed in Refs. 178–179. An approach within the LCSR has also been formulated to explore the S-wave generalized form factors for the heavy meson transitions into the $\pi\pi$, $K\pi$ final states.¹⁸⁰

LHCb has recently reported the most precise measurement of the branching ratio for the $B^+ \rightarrow K^+ \mu^+ \mu^-$ channel to date, together with a study of its angular distribution and differential branching fraction.¹⁸¹ In the SM, the differential decay rate can be written as

$$\frac{1}{\Gamma} \frac{d\Gamma(B^+ \rightarrow K^+ \mu^+ \mu^-)}{d\cos\theta} = \frac{3}{4} (1 - F_H) (1 - \cos^2\theta) + \frac{1}{2} F_H + A_{FB} \cos\theta \quad (31)$$

where θ is the angle between the μ^- and the K^+ in the rest frame of the dimuon system. The two parameters, F_H and the forward-backward asymmetry of the dimuon system, A_{FB} , depend on q^2 . In the SM, A_{FB} is zero and F_H highly suppressed, and their measured values are consistent with the SM expectations. The differ-

ential branching fraction of the $B^+ \rightarrow K^+ \mu^+ \mu^-$ decay is, however, consistently below the SM prediction at low q^2 .¹⁸¹ The measurement of the CP asymmetry $B^+ \rightarrow K^+ \mu^+ \mu^-$ decay, instead, is consistent with the SM predictions.¹⁸² A broad peaking structure is observed in the dimuon spectrum of the same decay, in the kinematic region where the kaon has a low recoil against the dimuon system; the contribution of the resonant decay and of the interference of the yield for dimuon masses seems larger than theoretical estimates.¹⁸³ LHCb reports also the actual more precise determinations of A_{FB} for the decay $B^0 \rightarrow K^{*0} \mu^+ \mu^-$.¹⁵⁵ More recently, LHCb has announced a 3.7σ local discrepancy in one of the q^2 bins for one of the angular observables.¹⁸⁴ This analysis has prompted a large number of theoretical investigations, searching for NP in the framework of the Randall Sundrum mode,¹⁸⁵ Z' new coupling,^{186, 187, 188, 189} MSSM,^{190, 191} Minimal Flavour Violation,¹⁹² models based on the gauge group $SU(3)_c \times SU(3)_L \times U(1)_X$,¹⁹³ and so on.

4. Conclusions

The experimental progress in semi-leptonic decays in the last years is impressive and the theoretical situation is rich in perspective. The perturbative calculations, in general, have reached a phase of maturity, and the larger theoretical errors are due to non-perturbative approaches.

Still awaiting firmly established solutions are a few dissonances within the SM, such as the so-called “1/2 vs 3/2 puzzle” and “gap problem”, the possibility of flavour violation in decays into tauons, the long standing tension between the inclusive and exclusive determination of $|V_{cb}|$ and $|V_{ub}|$, and the recent $B \rightarrow K^* \mu^+ \mu^-$ anomaly.

The experimental analysis performed with the full Babar data sample for semileptonic $B \rightarrow D^{(*)} \tau \nu_\tau$ seems to confirm the excess with respect to the SM. Several NP scenarios have been devised, while on the experimental side it would be interesting to compare with results from the full Belle data sample.

Inclusive and exclusive values of $|V_{cb}|$ of comparable precision are available, although, on the whole, inclusive decays remain more precise. Their tension seems strengthened from the most recent lattice calculation and confirmation from different lattice groups would be welcome. The discrepancy between the values of $|V_{ub}|$ is larger, and the uncertainty on this value propagate on other $|V_{ub}|$ dependent observables (see e.g. Ref. 194). The error on the inclusive determination, around 4%, is about one half than the one on the exclusive determinations. Lattice computations that are now in progress may help in the near future to reduce the exclusive uncertainties. By increasing statistics, other channels, as $B \rightarrow \rho/\omega l \nu$, but also baryonic ones, can become an interesting alternative to the traditional $B \rightarrow \pi l \nu$ decay for the exclusive extraction of $|V_{ub}|$.

Recently, the most active field has been the one related to rare decays, prompted by the latest measurements of the angular correlations of the decay products in $B \rightarrow K^* \mu^+ \mu^-$, that display several deviations from the SM predictions, the largest

being around 3.7σ . Rare decays have always been the search ground of election for NP in flavour physics, and further data and analyses are longingly awaited to ascertain if this channel shows real departures from SM expectations or they result from statistical fluctuations and/or underestimated error uncertainties.

Acknowledgments

The author acknowledges partial support by Italian MIUR under project 2010YJ2NYW and INFN under specific initiative QNP.

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